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THE VALUE OF TOPOLOGICAL CONCEPTS TO THE BASIC UNDERSTANDING OF FORM AND SPACE.

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Abstract

Introduction
Topology is a branch of mathematics dealing with spatial relationships. Such a relationship possesses an elasticity not readily discernible which, however, touches upon a profound level of human perception. A basic topological relationship tends to remain unaffected by simple formal variations retaining its peculiar character. Although being a quite abstract subject, topology lies at the core of the mode in which ordinary perception operates. In basic pattern recognition, for instance, it becomes evident that observers focus not on metric but on topological features. Perception receives sensations of topological nature before these synthesize into a particular form. These sensations are found at the base of the development of character and when appropriately handled can direct the unconscious toward a desired meaning. Alternatively, topological aspects of form and space can give rise to new sensations that generate unplanned character or meaning.

Materials and Methods
Since topological attributes appear to be so fundamental in perceiving and interpreting form and space it is imperative that students in the design disciplines became acquainted with them early on in their training in order to be enabled to generate character and meaning through form. Topological attributes such as proximity, directionality, enveloping, crossing, arouse sensations and determine kinds of form that might serve the intended purpose.
The methods used to help students become familiar with such an abstract manner of perceiving focus on simple exercises of an analytical nature. Works of art or architecture are freely chosen and graphically analyzed in respect to recognizable relationships considered as fundamental to their expression. The student is asked to elucidate simple arrangements through basic gestures attempting to capture the fundamental character of the art work. Also, certain compositions are presented of which the essential character is to be discerned by the student in the dynamic relationships developed among materials or objects and expressed in a drawing or collage. Gradually, the analytic yields to the synthetic mode in exercises in which the student is asked to respond to a compositional need. Before composing a solution to a given problem he is asked to devise the most appropriate gestures in response to it, expressing the fundamental topological character that ought to be embodied in the potential forms.

Results
The results may be kinesthetic expressions whether gestures or movements of the entire body or small projects whether collages, drawings, or models representing an expressive aspect of relationships seen in an arrangement of forms or intended as a solution to a problem by the student.

Conclusions

These exercises appear to be rewarding in the sense that they liberate the intellect of the student and place it in closer contact with the realm of feelings and the unconscious indicating the fundamental role of topological concepts in common perception and the assignment of meaning.

Keywords
Topology, basic design, space, form, character

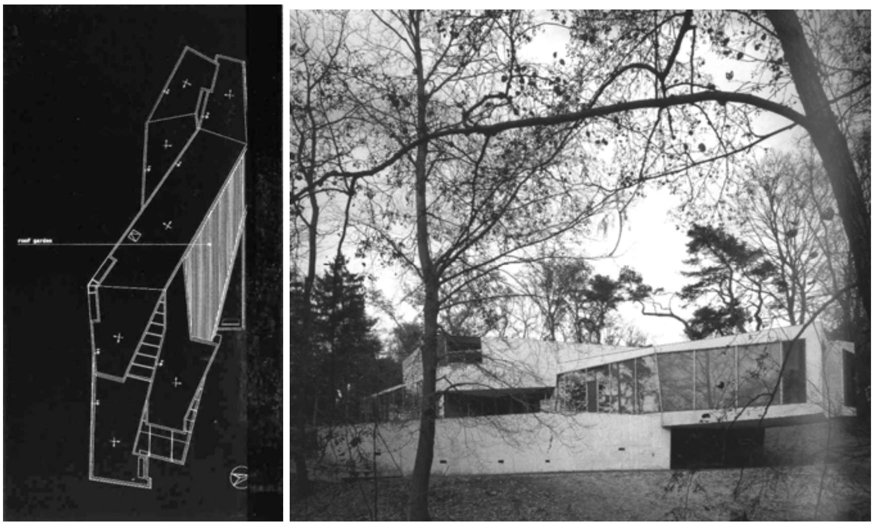
Introduction

Topology is a branch of mathematics which deals with spatial relationships. One of its founders was the German mathematician Auguste Ferdinand Möbius (1790-1868) who, in 1865, invented the homonymous strip (Fig. 1), a regular band which he cut, turned one end by 180º and reattached it to the other. From a topological point of view this was an achievement because through this action a fundamental transformation of relations took place. The initial two faces of the band had now fused into one and the formerly exclusive relationship of one face with the interior space and of the other with the exterior had now turned into a relationship of a single surface with the space which could be thought as either interior or exterior.

Figure 1. The Möbius Strip

In general, topology studies those properties of geometric forms that remain invariant when certain transformations take place, such as bending, stretching, etc. It is a part of set theory and was developed in the first decades of the twentieth century. The development of set-theoretic methods in topology led to a substantial extension of our idea of space (Alexandroff, 1961, p.3, 5).

In recent days seems that topology has exerted a considerable impact on contemporary architecture as may be observed in the Möbius house designed by UN Studio in 1993 near Amsterdam (Fig. 2, 3) which follows the form of the aforementioned strip.



Figures 2, 3. Möbius house, Amsterdam, 1993, plan and exterior

However, the importance of topology lies not in the potential imitations of its geometrical constructs, regardless of how sophisticated such imitations may be, but rather in the way in which it expands and cultivates our idea of space and does so indeed in a rather drastic manner.

The topological idea of a space depends not on the nature of the objects found in it but on the relations among them (Alexandroff, 1961, p.9). Hence, it impels us to overcome the

conception of geometry as an accumulation of rigid forms, in the Platonic and Euclidean sense, introducing the relationships between objects as a more deeply rooted factor than form *per se*.
The normal standpoint holds that what meets the eye is nothing but form, and more specifically, its outline, but it appears that perception actually deals mostly with sensations of topological nature. Such sensations tend to be unaffected by simple formal variations and deformations while they retain their peculiar character. A topological relationship exhibits some kind of elasticity not readily noticeable whereas it touches upon a profound level of human perception. Its intuitive comprehension requires the cooperation of both the visual and kinesthetic senses. Thus, even though it is a quite abstract subject, topology lies at the core of the mode in which ordinary perception operates. In basic pattern recognition, for instance, observers focus not on metric but on topological features. This aspect of topology is clearly understood by mathematicians. An early specialist in topology, states: “The significance of topology lies in the fact that its most important questions and theorems have an immediate intuitive content and thus teach us in a direct way about space” (Alexandroff, 1961, p.1).
Topology, however, may not be thought as being of assistance only in practical aspects of life such as the identification of basic patterns. The sensations aroused by topological relations lie at the base of the conception of character and meaning, and when appropriately handled may direct the unconscious toward any such desired end.

The nature of the relational structure and the operation of perception.
As mentioned above, any pattern recognition is based not on the metric properties of form but rather on qualitative characteristics of relative location or size a fact which is revealed on the concentration on topological features when observers are asked to draw what they see (Arnheim, 1990, p.113). The observer can go through exercises which lead him to perceive the similarity or difference of relationships occurring among different patterns (Fig. 4). He must discover in these the essential structure of a pattern before he can relate it to a second pattern and by comparison perceive similarity or difference between the two.

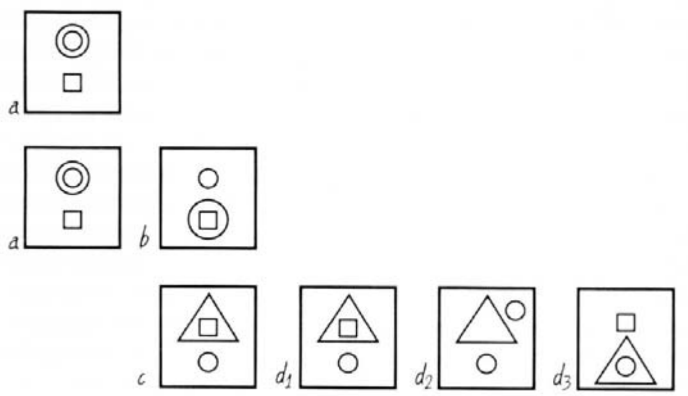


Figure 4. Recognition of relationships

A topological relationship between two patterns refers to the discovery of an analogous relationship as opposed to a mere object similarity. Relational analogies may be discovered between different patterns regardless of whether they contain similar shapes or not. The manner in which shapes relate to each other in one pattern may be identified again in another, employing entirely different shapes. Such identification, however, shall take place only when the essential relational structure or topological character is comprehended (Arnheim, 1990, pp.113-4). For instance, the topological similarity between *a* and *b* is not found between *c* and

d_1 nor *c* and d_2 but is present between *c* and d_3. It appears, therefore, that the ability to perceive topological structure is both more abstract and fundamental than the ability to perceive outer form but at the same time it intensifies form awareness. Topological structure may also be thought as a kind of form but of a different nature. The clearly metric relationships must be disregarded in order for the hidden, underlying relationships to become visible. One must, for a moment, shut his eyes to plain form in order to perceive deeper form or, more accurately, make a note of the various forms, turn his attention away from them and then delve deeper and attempt to see with his mind's eye the essential relational structure between forms.

The human mind is adapted to perform precisely such functions, that is, to work out such topological characteristics providing information about the typical character of things rather than their particular metric dimensions. Quantitative information is of no assistance to the perception of topological criteria (Arnheim, 1990, p.114). The analytical mathematical formula of a geometrical shape, for example, the circle, gives the location of all the points of which the circle consists. It does not describe its particular character, its central symmetry, its rigid curvature etc. It is precisely this perception of the character of a given phenomenon that renders productive thinking possible. All intelligence tests abound with analogies because analogies are better detected by one able to perceive the basic similarity of character between things he compares and intelligence is revealed in the way in which it perceives (Arnheim, 1990, p.117). In mathematics, a topological statement or drawing defines a spatial relationship, such as something that is contained or overlapped, with utmost accuracy although it may leave the real shapes entirely undefined, an approach which may lead to the sharpening of the idea (Arnheim, 1990, pp.153-4). Even the early children's drawings are of topological rather than geometrical nature, that is, they aim at general, non-metric properties such as the round, the closed, the rectilinear, rather than at specific, ideal embodiments (Piaget, 1967).

Topological relations precede form generation.

A topological space, in mathematics, is nothing other than a set of arbitrary elements, called points, in which a concept of continuity is defined. This concept of continuity is based on the existence of relations, which may be defined as local or neighborhood relations and it is precisely these relations which are preserved in a continuous mapping of one figure onto another (Alexandroff, 1961, p.8), in other words, the preservation of these relations is accountable for the occurrence of identification or recognition. I will present here an example of a topological problem which indicates its fundamental character and the manner in which it can give rise to form. This problem is called the *Jordan Curve Theorem* (Alexandroff, 1961, p.2). It refers to a simple closed curve which lies in the plane and divides it into two regions, itself serving as their common boundary. What matters to its topological nature is that it distinguishes the plane into two regions, the inside and the outside. In the two dimensional plane there are pairs of points so that a straight or polygonal line connecting them will necessarily intersect with the curve (Fig. 5).

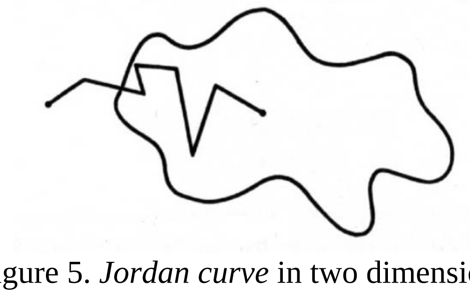


Figure 5. *Jordan curve* in two dimensions

Such pairs of points are separated by the curve or "linked" by it. Now, let us consider the *Jordan Curve* in three-dimensional space. In this case there are no pairs of points which are separated by the curve because there would always be a line either straight or polygonal that would pass around the curve and connect the two points. Closed polygons would be formed which are linked with the curve in a specific way, that is, their surface would intersect with the curve (Fig. 6).

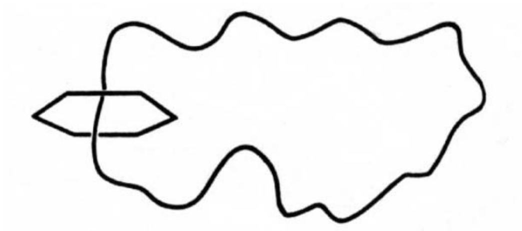


Figure 6. *Jordan curve* in three dimensions

Here, a substantial qualitative change has taken place once we moved from the two-dimensional plane to three-dimensional space. A simple relation of two intersecting shapes took on an entirely different character. It was transformed into a threaded relationship. Intersection was transformed into threading or perhaps the intensity was put more on separation in the first case while in the second case it tipped more toward linking. Still, the question of form is pending. Any number of forms could fulfill this relationship. It is imperative though to understand that if this relationship were not clarified at this more abstract level it would be quite difficult to be apprehended by a student who would have dealt with specific forms from the outset. The apprehension of outer form and even its skillful handling would have impeded or, at best, delayed the penetration of the student's mind into the fundamental concept which could then be applied to the structure of a bracelet as well as to that of a freeway system. The reason for this difficulty lies on the fact that, as already observed in antiquity, once perceived, form tends to impress itself upon memory and displace other deeper or subtler aspects. This was also an observation made by 20^{th} century artists about representational form who, in their effort to break away from the power of that impression, invented abstract art.

The expansion of spatial imagination

A relationship, such as the one described by the *Jordan Curve Theorem* even though it starts from a common mathematical definition leads the imagination to perform a leap in order to reach its three dimensional expression. Of interest here is what one may learn about spatial relationships and how one can benefit from it in the design disciplines. To begin with, students need to expand their spatial imagination. They must learn through such exercises in what ways relationships are transformed or remain unchanged as we expand into space. In the *Jordan Curve* example, all at once a sense of separation and of the presence of a controlling agent between two positions, occurring in two dimensions, is overcome by an imaginative expansion of the same principle into three dimensions. The student may realize that he can dig below or rise above the closed curve in order to reach to the other side and touch or see one of the two points.

The shape of the curve is not really important as long as he performs this motion of getting around it. As long as he has trodden this path he realizes that he has generated a loop around the line of the curve. His path now is inextricably linked with the curve in a peculiar way similar to that of a rosary or a string of beads. The beads, however, need not be similar nor the curve smooth, thereby reaching a point of defining a set of relationships which do not display a definite form but rather pre-formal conditions or principles which may be satisfied by numerous forms. Such principles characterize the *sub-phenomenal* aspect of what is looked at, i.e. the condition underlying the appearance of an image which determines to a large extent both its formal and kinesthetic outward expression. The desire to form is more sensible to begin with the resolve to determine spatial relationships before proceeding to the final outline of shape. Such relationships are often unconsciously tinkered with in sketches during the process of resolving a design problem. However, an insight into the topological nature of such problems is of immense significance to design students.

Topological spaces and derivative concepts

As will be seen below, within every topological problem there are certain basic concepts pertinent to them. As already said a topological space is derived once all distance functions of a metric space have been discarded and only the relationships are retained (Mendelson, 1990, p.70). A topological space is a set in which certain subsets are defined and are associated to the points of the space as their neighborhoods. Depending upon the axioms these neighborhoods satisfy, one distinguishes between different types of topological spaces. Consequently, there is equivalence between the concept of neighborhood and topological space (Mendelson, 1990, p.70). A neighborhood is a subset or an area in which some kind of proximity exists. Such proximity, however, does not refer to distance but to an underlying rule by virtue of which a concept of continuity is established (Mendelson, 1990, p.75-6).

The most important among the topological spaces are the so-called Hausdorff spaces (Alexandroff, 1961, pp.8-9). The Hausdorff spaces are occasionally called “separated” spaces because they contain neighborhoods of points clearly separated from each other (Mendelson, 1990, p.77). In such spaces we have sets of points clearly distinguishable from each other that, within each neighborhood, present common characteristics. Since their similarity depends only on the relations and not on the nature of the respective objects (Alexandroff, 1961, p.9) the concepts of neighborhood and continuity are established among objects based on the constancy of their relationship. Several other topological concepts exist such as closure, interior, boundary (Mendelson, 1990, p.71), crossing, touching, surrounding etc. which remain unchanged by deformations and constitute structural features that affect the recognition process (Arnheim, 1990, p. 60).

Teaching topology to the design disciplines

It would certainly be desirable for design students to possess knowledge of topology from a mathematical perspective, which, however, may not usually be expected. Once the advantage of such knowledge has been established the method of teaching topological principles to students with an often limited scientific background remains in doubt. On the one hand, it should be ideal to set up joint courses on mathematics and design in the University curriculum, among which a course in topology and its impact on design issues might be included. However, this would certainly constitute a long term goal which would necessitate cautious development. A short term goal might be to approach the matter in a somewhat simplified manner attempting to popularize the subject matter. This could be achieved by explaining the fundamental concept in plain terms and contrive exercises through which the student might explore it by artistic means or inversely, give cautiously planned exercises and then attempt to make the students see the topological concept incorporated into their projects. This last approach, in its two facets, is the one I have recently taken in design theory courses.

Exercises are designed to help students become familiar with such an abstract manner of perceiving focusing initially on strengthening their analytical capacities which are subsequently applied in composition. Works of art are freely chosen and analyzed in respect to relationships considered to play an essential role in their expression. Students are asked to interpret and express simple arrangements initially through basic gestures attempting to capture the fundamental character of the percept. Subsequently, the essential character is to be discerned by the student in the dynamic relationships developed among materials or objects and expressed in a drawing or collage. In this process the topological relationships are pinpointed by the instructor. Slowly, the analytic mode gives way to the synthetic in exercises in which the student is asked to respond to a compositional need. Before composing a solution to a given problem he is asked to devise the most appropriate gestures in response to it, expressing the fundamental topological character that ought to be embodied in any shapes and forms potentially used.

Results

The results are kinesthetic expressions whether gestures or movements of the entire body or small projects whether collages, drawings, or models representing an expressive aspect of relationships seen in an arrangement of forms or intended as a solution to a problem by the student.


Figure 7, 8. Mutual influence of figure and ground. Exploration of density, depth and orientation of subspaces. A.Papanakli, 2006.

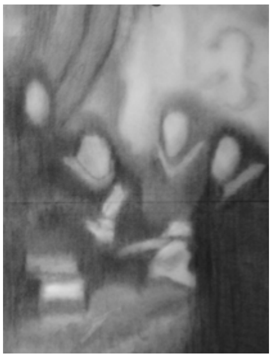



Figure 9, 10. Outlining meaning through chiaroscuro analysis. Exploration of relationships such as direction, density, and grouping of light areas and the manner in which they are combined to produce meaning. E. Vogdou, 2006.


Figure 11, 12. Sense of twirling through stroboscopic change of direction. The concept of neighborhood is established among light areas which then generate continuity because of their directional change. Yanna, 2006.

The results have thus far been satisfactory despite the short period of implementation of the method. Students have started developing an eye for relations between forms, between forms and spaces, and between spaces of different attributes. As a first step, the focus on relations has been achieved. The next task will probably be to study the various aspects or attributes of topological relationships such as neighborhood, continuity, connectedness, compactness etc., while a third task will be to derive meaning through basic analysis of relationships and of relative strengthening of their attributes.

Conclusions

Since topological concepts appear to be so fundamental in perceiving and interpreting form and space, students newly introduced to the design disciplines should certainly become acquainted with them early on in their training. The aforesaid method can form a teachable and comprehensible path through which the students might be enabled to reach an understanding of how to impart character and meaning to the forms and spaces they create. Topological relations such as proximity, directionality, enveloping, or crossing arouse sensations and by preceding the manifestation of concrete form they determine kinds of form that might serve the intended purpose. Carefully designed topological exercises appear to be rewarding in the sense that they liberate the intellect of the student and place it in closer contact with the realm of feelings and the unconscious not only indicating the fundamental role of topological concepts in common perception but also revealing a possible path toward artistic expression and the initiation of meaning.

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